2023 AP Daily: Practice Sessions

AP Statistics

Session 2 – FRQ (Part A: Probability)

1. A machine at a manufacturing company is programmed to fill shampoo bottles such that the amount of shampoo in each bottle is normally distributed with mean 0.60 liter and standard deviation 0.04 liter. Let the random variable *A* represent the amount of shampoo, in liters, that is inserted into a bottle by the filling machine.

    a. A bottle is considered to be underfilled if it has less than 0.50 liter of shampoo. Determine the probability that a randomly selected bottle of shampoo will be underfilled. Show your work.

After the bottles are filled, they are placed in boxes of 10 bottles per box. After the bottles are placed in the boxes, several boxes are placed in a crate for shipping to a beauty supply warehouse. The manufacturing company's contract with the beauty supply warehouse states that one box will be randomly selected from a crate. If 2 or more bottles in the selected box are underfilled, the entire crate will be rejected and sent back to the manufacturing company.

    b. The beauty supply warehouse manager is interested in the probability that a crate shipped to the warehouse will be rejected. Assume that the amounts of shampoo in the bottles are independent of each other.

        i. Define the random variable of interest for the warehouse manager and state how the random variable is distributed.

        ii. Determine the probability that a crate will be rejected by the warehouse manager. Show your work.

To reduce the number of crates rejected by the beauty supply warehouse manager, the manufacturing company is considering adjusting the programming of the filling machine so that the amount of shampoo in each bottle is normally distributed with mean 0.56 liter and standard deviation 0.03 liter.

    c. Would you recommend that the manufacturing company use the original programming of the filling machine or the adjusted programming of the filling machine? Provide a statistical justification for your choice.

2. A company manufactures model rockets that require igniters to launch. Once an igniter is used to launch a rocket, the igniter cannot be reused. Sometimes an igniter fails to operate correctly, and the rocket does not launch. The company estimates that the overall failure rate, defined as the percent of all igniters that fail to operate correctly, is 15 percent.

   A company engineer develops a new igniter, called the super igniter, with the intent of lowering the failure rate. To test the performance of the super igniters, the engineer uses the following process.

   - Step 1: One super igniter is selected at random and used in a rocket.
   - Step 2: If the rocket launches, another super igniter is selected at random and used in a rocket.
   - Step 2 is repeated until the process stops. The process stops when a super igniter fails to operate correctly or 32 super igniters have successfully launched rockets, whichever comes first. Assume that super igniter failures are independent.

   a. If the failure rate of the super igniters is 15 percent, what is the probability that the first 30 super igniters selected using the testing process successfully launch rockets?

   b. Given that the first 30 super igniters successfully launch rockets, what is the probability that the first failure occurs on the thirty-first or the thirty-second super igniter tested if the failure rate of the super igniters is 15 percent?

   c. Given that the first 30 super igniters successfully launch rockets, is it reasonable to believe that the failure rate of the super igniters is less than 15 percent? Explain.